Preface

This book is designed as an advanced undergraduate or a first-year graduate course for students from various disciplines and in particular from Economics and Social Sciences. It has evolved while teaching courses of Advanced Mathematics at the Bocconi University. The heterogeneous background of the students from different areas, has suggested an almost self-contained presentation. The result is a book divided in two parts.

In the *first part*, constituted by the chapters from 1 to 8, the fundamental aspects of mathematical modelling are developed, dealing with both continuous time systems (differential equations) and discrete time systems (difference equations). Particular attention is devoted to equilibria, their classification in the linear case, and their stability. An effort has been made to convey intuition and emphasize connections and concrete aspects, without giving up the necessary theoretical tools.

In the *second part*, from chapters 9 to 11, the basic concepts and techniques of Dynamic Optimization are introduced, covering the first elements of Calculus of Variations, the variational formulation of the most common problems in deterministic Optimal Control, both in continuous and discrete versions. Chapter 11 contains a brief introduction to Dynamic Programming.

To avoid heavy technicalities and to facilitate the understanding of the fundamental ideas, both state and control variables are one-dimensional. We believe that, once understood the one-dimensional case, the reader will be able to generalize the results to any number of dimensions without much effort, using the specialized books on the subject, listed in the references.

For the first part the preliminary requirements are limited to a knowledge of the Riemann integral and the multidimensional differential calculus, besides basic notions of linear algebra, briefly recalled in the Appendix.

The second part requires the knowledge of static (free and constrained) optimization for functions of several variables. The Fermat theorem, the method of Lagrange and Karush-Kuhn-Tucker multipliers are briefly recalled in the Appendix.

At the end of every chapter, a list of exercises is proposed, whose solutions can be found in the website: www.egeaonline.it.

Milan, January 2018 *The Authors*